

Unit 13

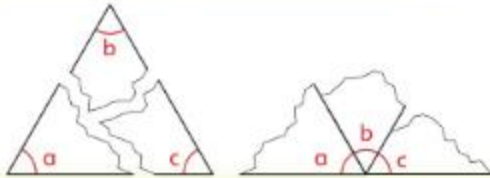
Geometry – properties of shapes



In this unit we will ...

- ✦ Measure angles and draw shapes accurately using a ruler and protractor
- ✦ Calculate unknown angles in shapes and on lines using angle facts
- ✦ Explore properties of polygons and circles
- ✦ Identify 3D shapes from 2D representations
- ✦ Draw multiple nets for a 3D shape

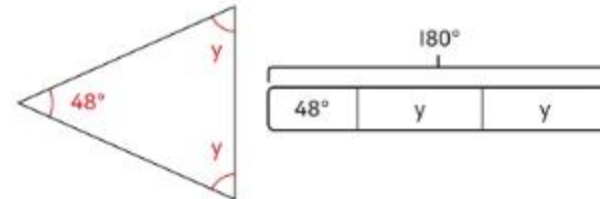
How can you use your knowledge of angles on a straight line to work out what the interior angles of a triangle add up to?



We will need some maths words. Which ones do you recognise? What do they mean?

degree angle obtuse acute reflex
 right angle protractor triangle isosceles
 equilateral scalene regular polygon quadrilateral
 parallelogram kite rhombus trapezium diameter
 radius circumference concentric perimeter
 net pyramid tetrahedron cylinder prism
 vertically opposite angles cuboid cube

We also need to be able to use bar models to calculate unknown angles. How can you work out the size of this angle without measuring?




Angles in polygons 1

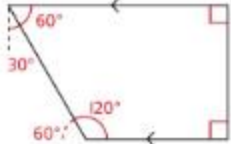
Discover

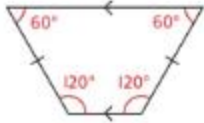


- 1** a) What shape is in step 1?
 What shape is left after completing step 2?
 What shape is left after completing step 3?
 Describe the properties of each shape as fully as you can.
- b) What shapes can be made by joining the two left-over triangles?

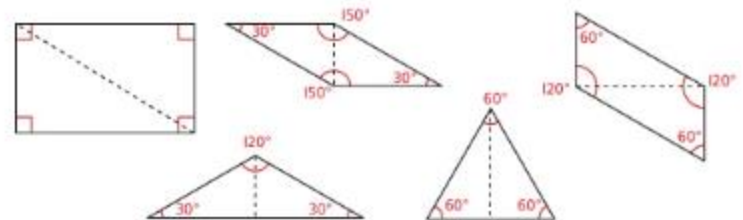
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a)  Step 1 is a rectangle. It has two pairs of parallel sides, and four right angles.
 The angles total: $90^\circ + 90^\circ + 90^\circ + 90^\circ = 360^\circ$

 After step 2, the shape is a right-angled trapezium. It has one pair of parallel sides, two right angles and sides of different lengths.
 The angles total: $60^\circ + 120^\circ + 90^\circ + 90^\circ = 360^\circ$

 After step 3, the shape is an isosceles trapezium. It has two pairs of equal angles and one pair of parallel sides.
 The angles total: $60^\circ + 120^\circ + 60^\circ + 120^\circ = 360^\circ$

b) The left-over triangles can form three quadrilaterals and two triangles.

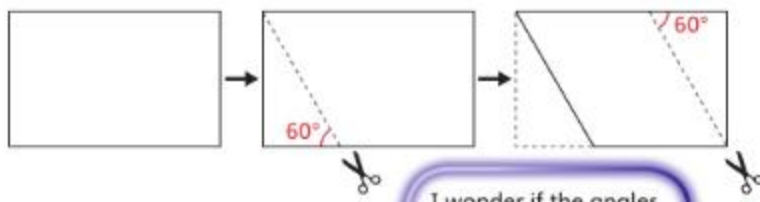


The angles in the quadrilaterals add up to 360° .

The angles in the triangles add up to 180° .

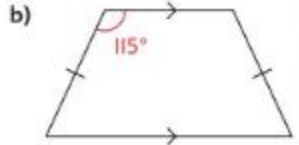
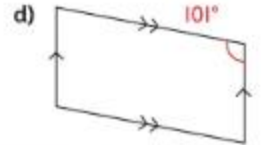
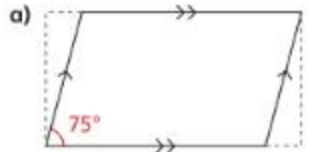
Think together

1 Describe the shape that is left after following these steps, including the total of all the angles.

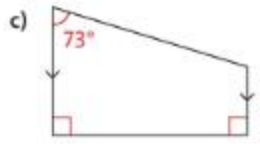


I wonder if the angles always add up to the same number.

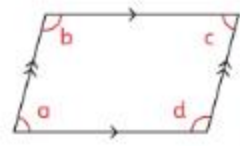
2 These shapes have been made from a rectangle. Find all of the angles in each shape.



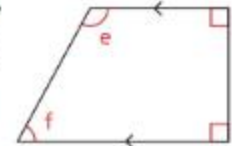
Remember, the lines with one arrowhead are parallel to each other. The lines with two arrowheads are also parallel to each other.



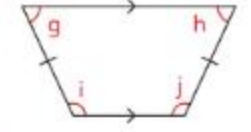
3 Explore the relationships in these shapes.



$$\begin{aligned}
 &a + b \\
 &c + d \\
 &b + c \\
 &a + d \\
 &a + b + c + d
 \end{aligned}$$



$$\begin{aligned}
 &f + e \\
 &f + e + \square + \square
 \end{aligned}$$

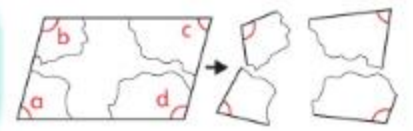


$$\begin{aligned}
 &g + i \\
 &h + j \\
 &g + j \\
 &h + i \\
 &g + i + h + j
 \end{aligned}$$

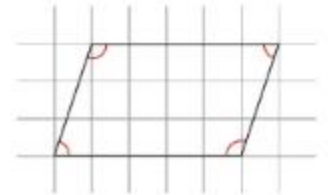
What do you notice?



I will create paper versions of each shape, and tear off the corners to investigate the angles.

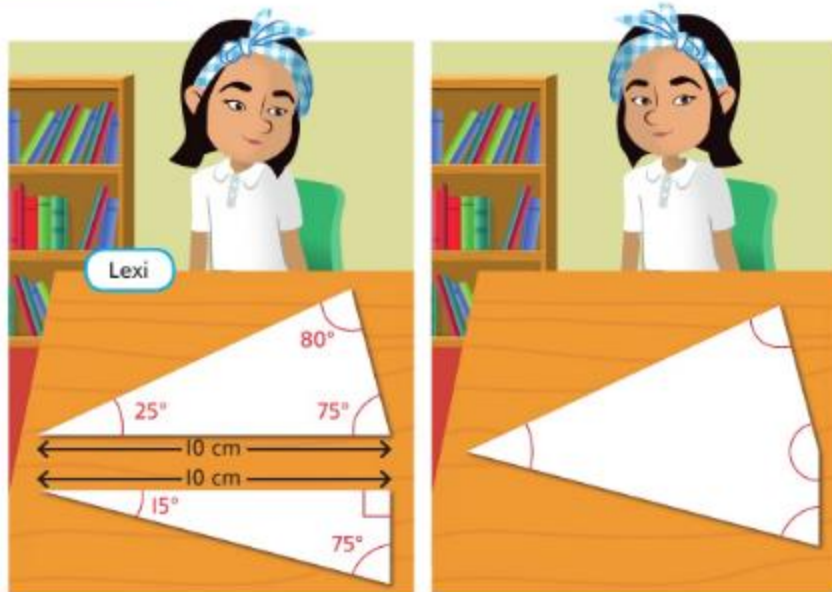


I wonder how I can add the angles if I do not know how many degrees they are. I will draw some grids to help me work it out.



Angles in polygons 2

Discover

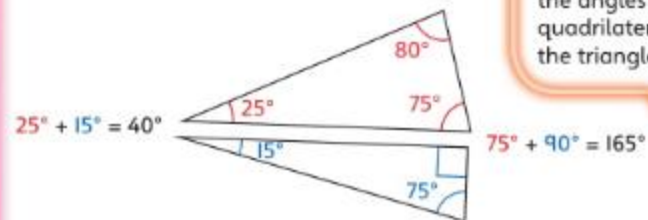


Lexi

- 1 a) What are the angles in the quadrilateral Lexi has created?
- b) Lexi says, 'I wonder if every quadrilateral has the same angle total.' What do you think? Explain your ideas.

Share

- a) The new quadrilateral is made from two triangles.



You can calculate the angles in the quadrilateral by adding the triangles' angles.



Lexi's quadrilateral has an angle total of $40 + 80 + 165 + 75 = 360^\circ$.

- b) Any quadrilateral can be split into two triangles.



These angles total 180° .



These angles total 180° .



$180 \times 2 = 360^\circ$

So the total of all the angles in a quadrilateral must be 360° .

Think together

1 Calculate the missing angles in the quadrilaterals below.

2 Max has made a kite and a rhombus on his geoboard.

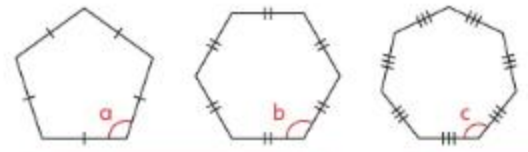
For both shapes, how many angles do you have to measure before you can calculate the rest of the angles?

3 Complete the table below.



Shape	Number of sides	Number of triangles	180 × number of triangles	Sum of interior angles
Square	4	2	180 × 2	360°
Pentagon	5	3	180 × 3	540°
Hexagon				
Heptagon				

Use this information to work out the angles in these regular polygons.



A regular shape must have all angles of equal size and all sides of equal length.

I think I could use this method to find the angles of any regular polygon.



Vertically opposite angles

Discover



Computing angle sizes. This may take a few minutes.

Experiment 1

Experiment 2

Experiment 3

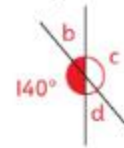
	a	b	c	d
Experiment 1	50°	130°	50°	130°
Experiment 2	60°	120°	60°	120°
Experiment 3	140°			

- 1 a) Complete the measurements for experiment 3.
- b) Lexi notices a pattern in the results. Some of the angles always seem to be equal. Can you explain this?

Share

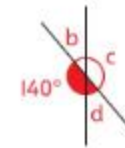
- a) Angles on a straight line add up to 180°. We can use this to calculate the remaining angles.

Angle a is 140°.



$$140 + b = 180^\circ$$

So b must be 40°.



$$140 + d = 180^\circ$$

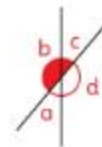
So d must also be 40°.



$$b + c = 180^\circ$$

b is 40° so c must be 140°.

- b) Pairs of opposite angles where two straight lines cross must always be equal. These are called **vertically opposite angles**.



$$b + c = 180^\circ$$

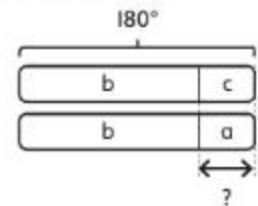


$$b + a = 180^\circ$$

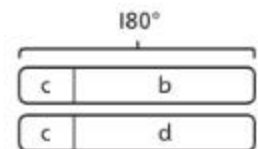
$$\text{So } a = c$$



$$b = d$$

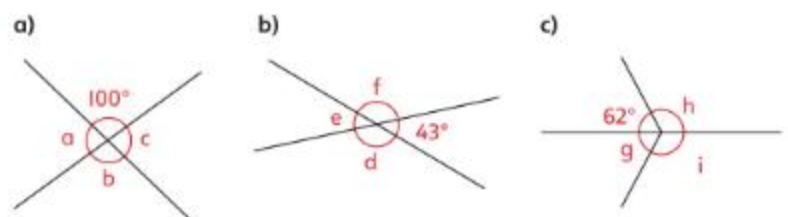


So a and c must be equal because b is the same each time.



Think together

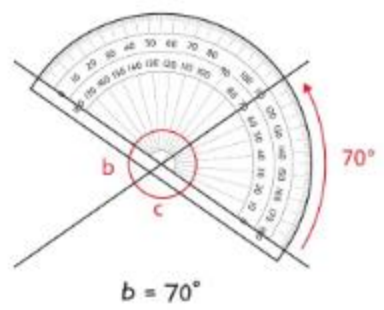
1 Calculate all the angles that you can.



I am not sure all of these are possible.



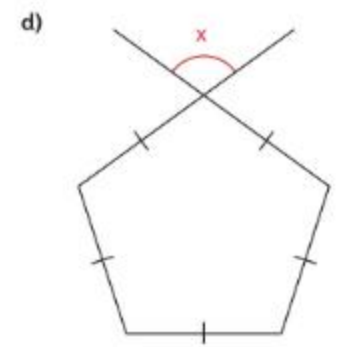
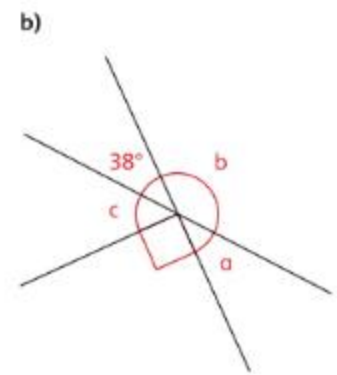
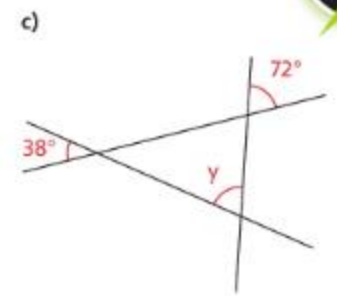
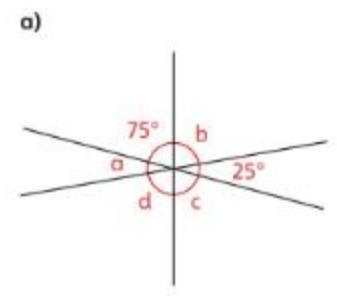
2 Draw pairs of straight lines that cross at different angles. Check that the opposite angles are equal by measuring with a protractor.



I measure one angle, then predict the rest of the angles before I check with a protractor.



3 Calculate the missing angles.



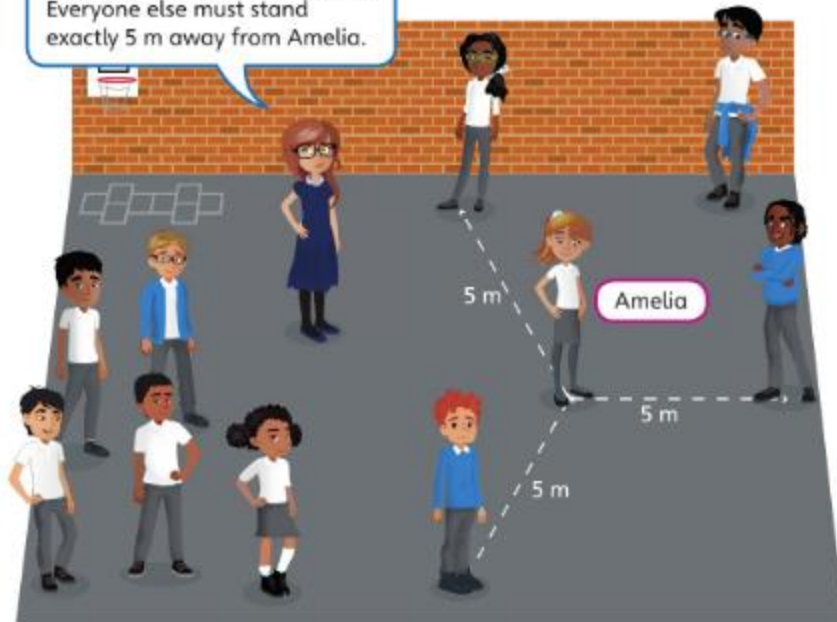
Discuss the order in which you worked out the angles.

Equal distance

Discover



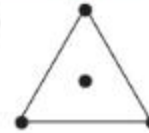
Amelia, you stand in the centre. Everyone else must stand exactly 5 m away from Amelia.



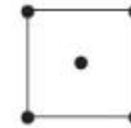
- 1 a) What shapes will the class form around Amelia?
- b) What would it look like if another class came and stood 6 m away from Amelia?

Share

a)



Three children will form a triangle around Amelia.



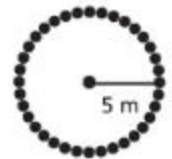
Four children will form a quadrilateral.



As more children join, the shape has more vertices.

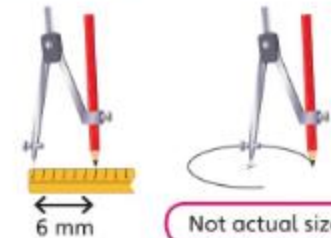


As more children join, they start to form a circle. The distance from the centre is called the **radius**. This circle has a radius of 5 m.



To draw smaller circles, you can use a pair of compasses.

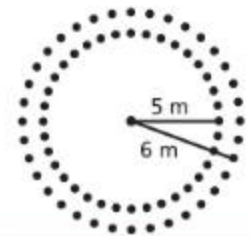
This pair would draw a circle with a radius of 6 cm.



Not actual size

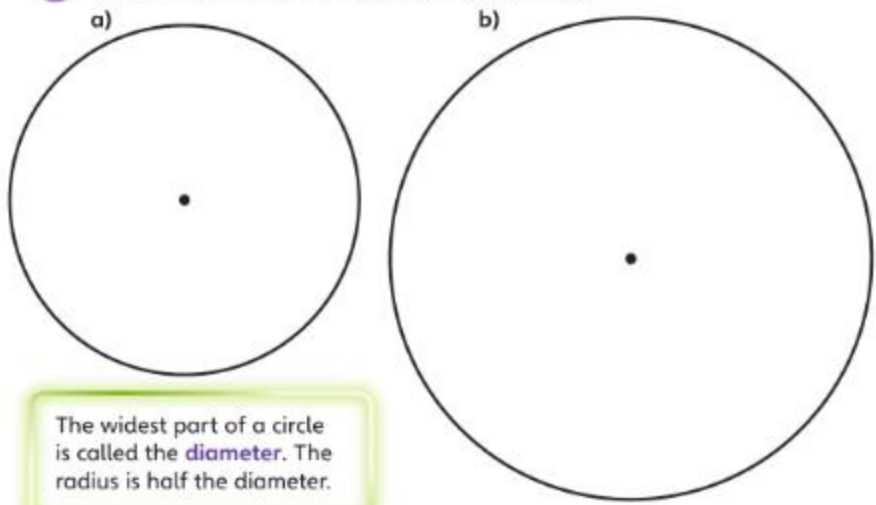
- b) The second class will form another circle. The radius will be 6 m, so it will be larger.

Circles with the same centre are called **concentric** circles.



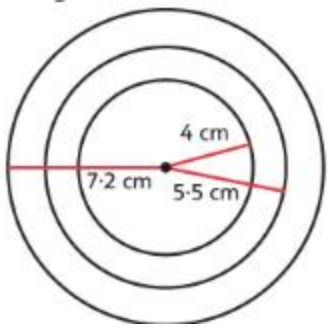
Think together

1 Measure the radiuses (radii) of the circles below.



The widest part of a circle is called the **diameter**. The radius is half the diameter.

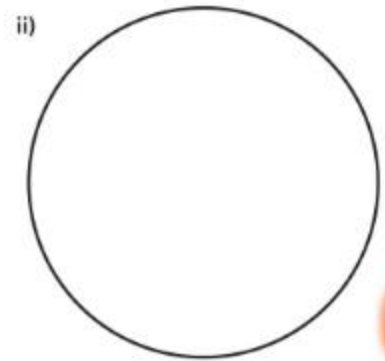
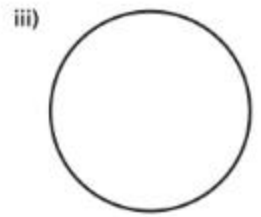
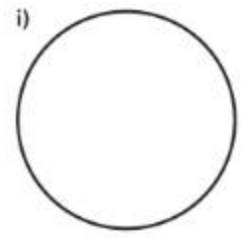
2 Try copying this design. If you do not have a pair of compasses, you could use string or a ruler to make the design.



Not actual size



3 Max has drawn some circles, but he forgot to mark the centres. Find the radius of each circle.



You can use the diameter to find the radius.



Check out this link for extra support

<https://www.bbc.co.uk/bitesize/topics/zvmxsbk/articles/z8c7qtv>

Parts of a circle

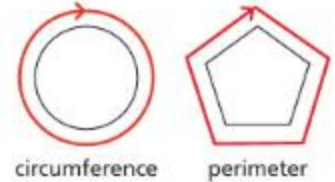
Discover



- 1 a) How could Bella find the distance around the edge of her bike's wheel?
- b) Which is longest: the diameter, the radius or the distance around the edge of the wheel?

Share

The **circumference** is the distance all around a circle. It is similar to the perimeter of polygons.



- a) There are different methods to measure the circumference of a wheel.

Method 1:

Wrap a piece of rope or string around the circumference, then measure the rope in a straight line.



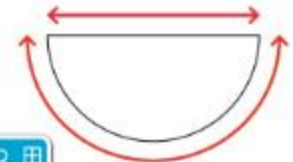
Method 2:

Choose a point on the wheel, and roll it until the point returns to the starting position. The distance it has rolled is the circumference.



- b) The radius is half the diameter, so the diameter is longer.

You can see in a semicircle that the curved part of half a circle is longer than the diameter.

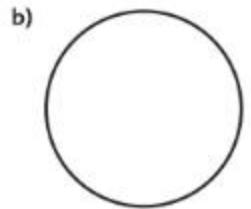
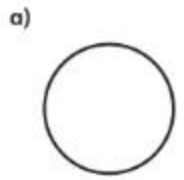


So the circumference of a circle, like the outside of a wheel, is always longer than the diameter.



Think together

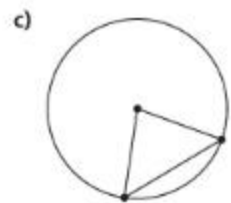
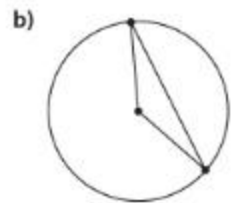
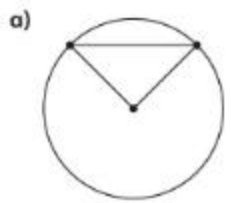
1 Estimate the circumference of each circle and then try to measure the circumference as accurately as you can.



I can use string to measure the circumference of these circles.



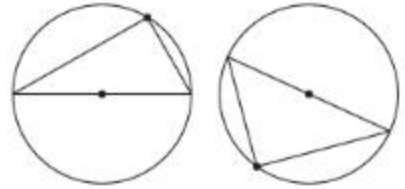
2 Bella chooses two points on the circumference of a circle and joins them to the centre to form different triangles.



What different types of triangle can Bella form?

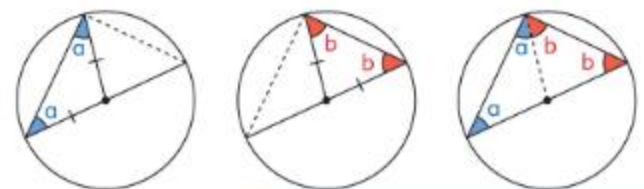


3 a) These triangles are formed by drawing a diameter through the centre, and joining each end of the diameter to a point on the circumference.



Measure the angles. What do you notice?

b) Isla has an idea about the size of the angle on the circumference. She has shown her reasoning in the diagrams below. Can you see what she has worked out?



I think I can see isosceles triangles. I wonder why.



So angles a and b join at the circumference. I think I know what they add up to. There are two a angles and two b angles in the triangle. I wonder what they add up to.

